

TOWARDS HIGHER- N SUPEREXTENSIONS OF BORN-INFELD THEORY

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We give a brief account of supersymmetric Born-Infeld theories with extended supersymmetry, including those with partially broken supersymmetry. Some latest developments in this area are presented. One of them is $N = 3$ supersymmetric Born-Infeld theory which admits a natural off-shell formulation in $N = 3$ harmonic superspace.

1. INTRODUCTION

The main source of current interest in superextensions of the Born-Infeld (or Dirac-Born-Infeld) action is the fact that these super BI actions constitute an essential part of the worldvolume actions of Dp -branes (see [1] and refs. therein). It is tempting to formulate super BI actions in a manifestly supersymmetric way using an off-shell superfield approach. In this approach the worldvolume SUSY is linearly realized, as opposed to the approach proceeding from a gauge-fixed form of the Green-Schwarz Dp -actions [2], in which all SUSYs are nonlinear and non-manifest. As was shown in [3], the $N = 1, d = 4$ super BI theory first formulated in [4] in terms of superfields can be re-interpreted as a theory of partial spontaneous breaking of global $N = 2$ SUSY (PBGS) down to $N = 1$ SUSY, with vector $N = 1$ multiplet as the relevant Goldstone multiplet. Thus the PBGS approach provides a powerful way of deriving super BI actions with extra hidden nonlinearly realized supersymmetries.

This contribution is a brief survey of super BI theories of this kind known to date. A short description of the recently constructed $N = 3$ supersymmetric BI theory [5] is also given.

2. SUPER BI THEORIES AS PBGS SYSTEMS

2.1 $N = 2 \rightarrow N = 1, d = 3$ BI theory. The basic object of this simplest super BI theory [6] is

the $N = 1, d = 3$ spinor gauge superfield strength $\mu^a(x^{bc}, \theta^d)$, $a, b, c, d = 1, 2$, subjected to the constraint

$$D_a \mu^a = 0, \quad (1)$$

where $D_a = \partial/\partial\theta^a + 1/2 \theta^b \partial_{ab}$. A nonlinear extension of the free $N = 1, d = 3$ Maxwell action, with the $d = 3$ BI action as the bosonic core, reads

$$S \sim \int d^3x d^2\theta w, \quad (2)$$

$$w \equiv \frac{1}{2} \frac{\mu^2}{1 + \sqrt{1 - D^2 \mu^2}}.$$

Besides being manifestly $N = 1$ supersymmetric, this action possesses the hidden nonlinear SUSY

$$\delta_\eta \mu_a = \eta_a (1 - D^2 w) + \eta^b \partial_{ab} w. \quad (3)$$

The superfield μ^a is the Goldstone superfield associated with the spontaneous breaking of the $N = 2, d = 3$ SUSY,

$$\{Q_a, Q_b\} = \{S_a, S_b\} = P_{ab}, \quad \{Q_a, S_b\} = 0, \quad (4)$$

down to the $N = 1$ one generated by Q_a, P_{ab} . The Goldstone fermion ψ^a associated with the spontaneously broken generators S_a in the standard nonlinear realizations approach (with making use of the exponential parametrization of the Poincaré supergroup elements) is related to μ^a by

$$\psi^a = \frac{\mu^a}{1 - D^2 w}. \quad (5)$$

In components, (2) gives rise to a static-gauge form of the space-filling D2-brane action.

Now let us briefly outline how the super BI action (2) can be deduced. The derivation follows a generic method [7,8] which is also applicable to other PBGS cases [9,10].

The starting point is the appropriate linear realization of the considered PBGS pattern. It is obtained by embedding the $N = 1, d = 3$ Maxwell superfield strength μ_a into a linear $N = 2, d = 3$ multiplet. The latter should have such a transformation law under the S -supersymmetry that μ_a transforms with an inhomogeneous term $\sim \eta_a$ and so can be interpreted as the Goldstone fermion of linear realization.

The appropriate $N = 2, d = 3$ supermultiplet was proposed in [11] as a deformation of the $N = 2, d = 3$ Maxwell multiplet. The superfield constraints defining this deformed multiplet can be taken in the form [10]

$$\begin{aligned} \text{(a)} \quad & [(D)^2 - (D^\zeta)^2] W = -2i , \\ \text{(b)} \quad & D^a D_a^\zeta W = 0 , \end{aligned} \quad (6)$$

where $W(x, \theta, \zeta)$ is a real $N = 2, d = 3$ superfield (ζ_a is an extra Grassmann coordinate). The standard S -supersymmetry transformation law of W ,

$$\delta_\eta W = -\eta^a \left(\frac{\partial}{\partial \zeta^a} - \frac{1}{2} \zeta^b \partial_{ab} \right) W , \quad (7)$$

implies the following transformation laws for the irreducible $N = 1$ superfield components of $W(x, \theta, \zeta)$, $\mu_a \equiv -i D_a^\zeta W|_{\zeta=0}$ and $w \equiv i/2 W|_{\zeta=0}$,

$$\begin{aligned} \text{(a)} \quad & \delta_\eta \mu_a = \eta_a (1 - D^2 w) + \eta^b \partial_{ab} w , \\ \text{(b)} \quad & \delta_\eta w = \frac{1}{2} \eta^a \mu_a . \end{aligned} \quad (8)$$

It is easy to check that eq. (8a) is consistent with the Bianchi identity (1) (which is none other than eq. (6b)).

The additional homogeneously transforming $N = 1$ superfield $w(x, \theta)$ can be expressed in terms of the Goldstone-Maxwell one μ_a by enforcing nonlinear constraints the precise form of which is dictated by the generic method of refs. [7]-[10] applied to the given system.

As the first step, one defines superfields $\tilde{\mu}_a$ and \tilde{w} as finite S -supersymmetry transforms of μ_a and

w , with the transformation parameter η^a being replaced by $-\psi^a(x, \theta)$:

$$\begin{aligned} \tilde{\mu}_a &= \mu_a - \psi_a (1 - D^2 w) - \psi^b \partial_{ab} w \\ &\quad - \frac{1}{4} \psi^2 \partial_{ab} \mu^b , \\ \tilde{w} &= w - \frac{1}{2} \psi^a \mu_a + \frac{1}{4} \psi^2 (1 - D^2 w) . \end{aligned} \quad (9)$$

These quantities homogeneously transform with respect to the whole $N = 2, d = 3$ SUSY. Therefore, one can covariantly equate them to zero,

$$\tilde{\mu}_a = \tilde{w} = 0 . \quad (10)$$

From these covariant constraints one derives the equivalence relation between ψ^a and μ^a (5), as well as the relation

$$w = \frac{1}{4} \frac{\mu^2}{1 - D^2 w} . \quad (11)$$

These are precisely the equations postulated in [6] (up to a rescaling of w). They can be used to express w in terms of either ψ^a or μ^a

$$w = \frac{1}{4} \frac{\psi^2}{1 + \frac{1}{4} D^2 \psi^2} = \frac{1}{2} \frac{\mu^2}{1 + \sqrt{1 - D^2 \mu^2}} . \quad (12)$$

This composite superfield is just the Goldstone superfield Lagrangian density in the action (2).

The same superfield D2-brane action can be written in a manifestly $N = 2$ supersymmetric form as an integral over the whole $N = 2$ superspace, with either W^2 or $N = 2, d = 3$ Fayet-Iliopoulos term as the Lagrangian densities.

2.2 $N = 2 \rightarrow N = 1, d = 4$ BI theory. This case [3] corresponds to the partial breaking of $N = 2, d = 4$ SUSY,

$$\begin{aligned} \{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} &= \{S_\alpha, \bar{S}_{\dot{\alpha}}\} = 2P_{\alpha\dot{\alpha}} , \\ \{Q_\alpha, S_\beta\} &= 0 , \end{aligned} \quad (13)$$

down to the $N = 1, d = 4 \propto (Q, \bar{Q}, P)$, with a vector $N = 1, d = 4$ multiplet as the Goldstone one. In terms of the $N = 1, d = 4$ Maxwell superfield strength $W^\alpha(z), \bar{W}^{\dot{\alpha}}(z)$ ($z = (x^{\alpha\dot{\alpha}}, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$),

$$\bar{D}^{\dot{\alpha}} W^\beta = 0 , \quad D^\alpha W_\alpha + \bar{D}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} = 0 , \quad (14)$$

the Goldstone superfield action ($N = 1, d = 4$ BI action) reads

$$S \sim \int d^4 x_L d^2 \theta \phi ,$$

$$\begin{aligned}\phi &= W^2 + \frac{1}{2}\bar{D}^2 \frac{W^2 \bar{W}^2}{1 - \frac{1}{2}A + \sqrt{1 - A + \frac{1}{4}B^2}} , \\ A &\equiv \frac{1}{2} (D^2 W^2 + \bar{D}^2 \bar{W}^2) , \\ B &\equiv \frac{1}{2} (D^2 W^2 - \bar{D}^2 \bar{W}^2) .\end{aligned}\quad (15)$$

The equivalence relation between the canonical nonlinear realization Goldstone spinor superfields $\psi^\alpha, \bar{\psi}^{\dot{\alpha}}$ and $W^\alpha, \bar{W}^{\dot{\alpha}}$ is [10]

$$W^\alpha = \psi^\alpha \left(1 - \frac{1}{4} \bar{D}^2 \bar{\phi} \right) + \dots , \quad (16)$$

where \dots stand for terms with x -derivatives. The nonlinear SUSY acts as

$$\delta_\eta W_\alpha = \eta_\alpha \left(1 - \frac{1}{4} \bar{D}^2 \bar{\phi} \right) - i \bar{\eta}^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \phi . \quad (17)$$

In components, (15) describes the space-filling D3-superbrane in a static gauge.

The action (15) can be deduced [10] from the appropriate linear realization of the PBGS pattern $N = 2 \rightarrow N = 1, d = 4$, following the procedure similar to that applied in the $N = 2 \rightarrow N = 1, d = 3$ case above. The starting point is $N = 2, d = 4$ Goldstone-Maxwell multiplet in the $N = 2$ superspace $z = (x^{\alpha\dot{\alpha}}, \theta_i^\alpha, \bar{\theta}^{\dot{\alpha}i})$. It is defined by the following deformation [12] of the standard $N = 2$ Maxwell superfield strength constraints

$$\begin{aligned}(a) \quad & D^{ik} \mathcal{W} - \bar{D}^{ik} \bar{\mathcal{W}} = i M^{(ik)} , \\ (b) \quad & D_\alpha^i \bar{\mathcal{W}} = \bar{D}_{\dot{\alpha}i} \mathcal{W} = 0 .\end{aligned}\quad (18)$$

Here,

$$D_\alpha^i = \frac{\partial}{\partial \theta_i^\alpha} - i \bar{\theta}^{\dot{\alpha}i} \partial_{\alpha\dot{\alpha}} , \quad \bar{D}_{\dot{\alpha}i} = - \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}i}} + i \theta_i^\alpha \partial_{\alpha\dot{\alpha}} ,$$

$$D^{ik} = D^{\alpha i} D_\alpha^k , \quad \bar{D}^{ik} = \bar{D}_{\dot{\alpha}i} \bar{D}^{\dot{\alpha}k} ,$$

and $M^{ik} = M^{ki}$ is a triplet of constants which explicitly break the automorphism $SU(2)_A$ of $N = 2$ supersymmetry down to $U(1)_A$ and satisfy the pseudo-reality condition

$$\overline{(M^{ik})} = \epsilon_{in} \epsilon_{km} M^{nm} .$$

Now we pass to the $N = 1$ superfield notation by relabelling the Grassmann coordinates and spinor derivatives as

$$\theta_1^\alpha \equiv \theta^\alpha , \quad \theta_2^\alpha \equiv \zeta^\alpha , \quad D_1^\alpha \equiv D_\alpha , \quad D_2^\alpha \equiv D_\alpha^\zeta .$$

In order to have the off-shell S -supersymmetry (acting as ζ -supertranslations) spontaneously broken while the Q -supersymmetry unbroken, we choose the following frame with respect to the explicitly broken $SU(2)_A$

$$M^{12} = 0 , \quad M^{11} = M^{22} = m , \quad (19)$$

where m is a real constant. Like in the case of D2-brane it is fixed up to rescaling of W . A convenient choice is

$$m = -2 .$$

It will be also convenient to choose a basis in $N = 2$ superspace where the chirality with respect to the variable ζ^α is manifest

$$\bar{D}_\alpha^\zeta = - \frac{\partial}{\partial \bar{\zeta}^{\dot{\alpha}}} , \quad D_\alpha^\zeta = \frac{\partial}{\partial \zeta^\alpha} - 2i \bar{\zeta}^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} . \quad (20)$$

In this basis, constraints (18) imply the following structure of the superfield $\mathcal{W}(x, \theta, \zeta)$

$$\mathcal{W} = -\frac{i}{2} \phi + i \zeta^\alpha W_\alpha - i \frac{1}{2} \zeta^2 \left(1 - \frac{1}{4} \bar{D}^2 \bar{\phi} \right) ,$$

where ϕ and W_α are chiral $N = 1$ superfields

$$\bar{D}_{\dot{\alpha}} \phi = \bar{D}_{\dot{\alpha}} W_\alpha = 0 , \quad (21)$$

and the fermionic superfield W_α obeys the $N = 1$ Maxwell superfield strength constraint (14).

The S -supersymmetry transformation of the $N = 2$ superfield \mathcal{W}

$$\delta_\eta \mathcal{W} = - \left[\eta^\alpha \frac{\partial}{\partial \zeta^\alpha} + \bar{\eta}^{\dot{\alpha}} \left(\frac{\partial}{\partial \bar{\zeta}^{\dot{\alpha}}} + 2i \zeta^\alpha \partial_{\alpha\dot{\alpha}} \right) \right] \mathcal{W}$$

implies the following ones for its $N = 1$ superfield components ϕ and W_α

$$\begin{aligned}\delta_\eta \phi &= 2(\eta W) , \quad \delta_\eta \bar{\phi} = 2(\bar{W} \bar{\eta}) , \\ \delta_\eta W_\alpha &= \eta_\alpha \left(1 - \frac{1}{4} \bar{D}^2 \bar{\phi} \right) - i \bar{\eta}^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \phi , \\ \delta_\eta \bar{W}_{\dot{\alpha}} &= \overline{(\delta_\eta W_\alpha)} .\end{aligned}\quad (22)$$

The superfield W_α shows up an inhomogeneous shift $\sim \eta_\alpha$ in its transformation, so it is the Goldstone fermion of the linear realization of the considered $N = 2 \rightarrow N = 1, d = 4$ PBGS pattern (Goldstone-Maxwell $N = 1$ superfield).

After this one can start the algorithmic procedure of passing to the relevant nonlinear realization. It goes in full analogy with the $N = 2 \rightarrow N = 1, d = 3$ case. Firstly one constructs finite η -transformations of the superfields ϕ and W_α proceeding from the infinitesimal ones (22). As the next step, one replaces, in the transformed superfields, the transformation parameters by the original nonlinear realization Goldstone fermions, $\eta_\alpha \rightarrow -\psi_\alpha$, $\bar{\eta}_{\dot{\alpha}} \rightarrow -\bar{\psi}_{\dot{\alpha}}$. At last, one imposes the covariant constraints on the so defined superfields

$$\tilde{\phi} = \tilde{W}_\alpha = 0, \quad (23)$$

and obtains the appropriate relations between ϕ , W_α and ψ_α . After some algebra, one ends up with the simple relations

$$\phi = \frac{W^2}{1 - \frac{1}{4}\bar{D}^2\phi}, \quad \bar{\phi} = \frac{\bar{W}^2}{1 - \frac{1}{4}D^2\bar{\phi}}, \quad (24)$$

which, up to a rescaling of ϕ , are just those postulated in [3] and derived from the nilpotency condition in [13]. Their solution is the expression for ϕ in (15).

Note that the Goldstone superfield Lagrangian density for the $N = 2 \rightarrow N = 1$ PBGS in (15) can be equivalently rewritten in $N = 2, d = 4$ superspace in terms of the constrained $N = 2$ gauge superfield strength $\mathcal{W}, \bar{\mathcal{W}}$ as the Fayet-Iliopoulos term or as the kinetic term of $N = 2$ Goldstone-Maxwell multiplet.

2.3 $N = 4 \rightarrow N = 2, d = 4$ BI theory. The $N = 2, d = 4$ super BI action can be constructed in terms of the $N = 2$ Maxwell superfield strength $\mathcal{W}(z), \bar{\mathcal{W}}(z)$, $z = (x^{\alpha\dot{\alpha}}, \theta_i^\alpha, \bar{\theta}^{\dot{\alpha}i})$, defined by the constraints

$$\bar{D}_\alpha^i \mathcal{W} = D_i^\alpha \bar{\mathcal{W}} = 0, \quad D^{ik} \mathcal{W} = \bar{D}^{ik} \bar{\mathcal{W}}, \quad (25)$$

($i, k = 1, 2$), which is the $M^{ik} = 0$ version of the constraints (18). The simplest $N = 2$ BI action was proposed in [14]. It goes into the previously given $N = 1$ BI action after the appropriate reduction, but it does not possess a second nonlinearly realized $N = 2$ SUSY. So it cannot be interpreted as the Goldstone superfield action for the PBGS option $N = 4 \rightarrow N = 2$ in $d = 4$.

A recursive procedure of restoring the correct action was developed in [15,16]. The point of de-

parture is the following $N = 4, d = 4$ superalgebra

$$\begin{aligned} \{Q_\alpha^i, \bar{Q}_{\dot{\alpha}j}\} &= \{S_\alpha^i, \bar{S}_{\dot{\alpha}j}\} = 2\delta_j^i P_{\alpha\dot{\alpha}}, \\ \{Q_\alpha^i, S_\beta^j\} &= 2\varepsilon^{ij}\varepsilon_{\alpha\beta}Z. \end{aligned} \quad (26)$$

The basic Goldstone superfield supporting the 1/2 breaking of this $N = 4$ SUSY is a scalar one W, \bar{W} associated with the complex central charge generator Z, \bar{Z} . It obeys a nonlinear version of the constraints (25) and is related to $\mathcal{W}, \bar{\mathcal{W}}$ by a complicated equivalence redefinition. Up to the sixth order, the $N = 4 \rightarrow N = 2$ Goldstone-Maxwell superfield action and transformations of the central charge and second hidden SUSY (with the generators S, \bar{S}) read

$$\begin{aligned} \delta\mathcal{W} &= f - \frac{1}{2}\bar{D}^4(fA) + \frac{1}{4}\square(\bar{f}\bar{A}) \\ &\quad + \frac{1}{4i}\bar{D}^{i\dot{\alpha}}\bar{f}D_i^\alpha\partial_{\alpha\dot{\alpha}}\bar{A}, \end{aligned} \quad (27)$$

$$\begin{aligned} A &= \bar{\mathcal{W}}^2 \left(1 + \frac{1}{2}D^4\mathcal{W}^2\right), \quad D^4 = \frac{1}{48}D^{ik}D_{ik}, \\ f &= c + 2i\eta^{i\alpha}\theta_{i\alpha}, \end{aligned} \quad (28)$$

$$\begin{aligned} S_{bi}^{(6)} &\sim \int d\zeta_L \mathcal{W}^2 + \text{c.c.} + \frac{1}{2} \int dz \{ \mathcal{W}^2 \bar{\mathcal{W}}^2 \\ &\quad \times [2 + (D^4\mathcal{W}^2 + \bar{D}^4\bar{\mathcal{W}}^2)] \\ &\quad - \frac{1}{9}\mathcal{W}^3\square\bar{\mathcal{W}}^3 + O(W^8) \}, \end{aligned} \quad (29)$$

where dz and $d\zeta_L$ are measures of integration over the whole $N = 2$ superspace and its chiral subspace. In [16] the action was restored up to 10th order using some algorithmic iteration procedure. The full action contains the standard BI action and some disguised form of the Nambu-Goto action for two physical scalar fields. It is a gauge-fixed action of D3-superbrane in $D = 6$.

Let us say a few words on the aforementioned recursive procedure of restoring the full superfield action and its relation to the approach based on the linear realizations of PBGS exemplified in the previous two subsections.

As was proposed in [16], the linear Goldstone-Maxwell multiplet relevant to the case at hand is an infinite-dimensional linear multiplet of $N = 4$ superalgebra (26), with a non-trivial realization of the central charges Z, \bar{Z} . To be more precise, one embeds the $N = 2$ Maxwell superfield

strength \mathcal{W} into an infinite-dimensional $N = 4$ multiplet

$$\mathcal{W}, \bar{\mathcal{W}}, \mathcal{A}_n, \bar{\mathcal{A}}_n, \quad (n = 0, 1, 2, \dots), \quad (30)$$

where \mathcal{A}_n are chiral (otherwise unconstrained) $N = 2$ superfields,

$$\bar{D}_{\dot{\alpha}i}\mathcal{A}_n = 0, \quad D_{\alpha}^i\bar{\mathcal{A}}_n = 0. \quad (31)$$

The following transformations

$$\begin{aligned} \delta\mathcal{W} &= f - \frac{1}{2}\bar{D}^4(f\bar{\mathcal{A}}_0) + \frac{1}{4}\square(\bar{f}\mathcal{A}_0) \\ &\quad + \frac{1}{4i}\bar{D}^{i\dot{\alpha}}\bar{f}D_i^{\alpha}\partial_{\alpha\dot{\alpha}}\mathcal{A}_0, \quad (32) \\ \delta\mathcal{A}_0 &= 2f\mathcal{W} + \frac{1}{4}\bar{f}\square\mathcal{A}_1 + \frac{1}{4i}\bar{D}^{i\dot{\alpha}}\bar{f}D_i^{\alpha}\partial_{\alpha\dot{\alpha}}\mathcal{A}_1, \\ \delta\mathcal{A}_1 &= 2f\mathcal{A}_0 + \frac{1}{4}\bar{f}\square\mathcal{A}_2 + \frac{1}{4i}\bar{D}^{i\dot{\alpha}}\bar{f}D_i^{\alpha}\partial_{\alpha\dot{\alpha}}\mathcal{A}_2 \\ &\quad \dots\dots\dots \\ \delta\mathcal{A}_n &= 2f\mathcal{A}_{n-1} + \frac{1}{4}\bar{f}\square\mathcal{A}_{n+1} \\ &\quad + \frac{1}{4i}\bar{D}^{i\dot{\alpha}}\bar{f}D_i^{\alpha}\partial_{\alpha\dot{\alpha}}\mathcal{A}_n, \quad (n \geq 1) \quad (33) \end{aligned}$$

where the function f was defined in (28), close off shell both among themselves and with those of the manifest $N = 2$ supersymmetry just according to the superalgebra (26). The central charge (c, \bar{c}) transformations non-trivially act on this infinite tower of $N = 2$ superfields.

A good candidate for the chiral $N = 2$ Lagrangian density is the superfield \mathcal{A}_0 . Indeed, the “action”

$$S = \int d^4x d^4\theta \mathcal{A}_0 + \int d^4x d^4\bar{\theta} \bar{\mathcal{A}}_0 \quad (34)$$

is invariant with respect to the transformation (33) up to surface terms.

It remains to define covariant constraints which would express $\mathcal{A}_0, \bar{\mathcal{A}}_0$ in terms of $\mathcal{W}, \bar{\mathcal{W}}$, with preserving the linear representation structure (32), (33). Because of an infinite number of $N = 2$ superfields \mathcal{A}_n , there should exist an infinite set of constraints expressing these superfields through the basic Goldstone ones $\mathcal{W}, \bar{\mathcal{W}}$. The procedure of deducing this set of constraints was described in [16]. The first two constraints read

$$\phi_0 = \mathcal{A}_0 \left(1 - \frac{1}{2}\bar{D}^4\bar{\mathcal{A}}_0\right) - \mathcal{W}^2$$

$$- \sum_{k=1} \frac{(-1)^k}{2 \cdot 8^k} \mathcal{A}_k \square^k \bar{D}^4 \bar{\mathcal{A}}_k = 0,$$

$$\begin{aligned} \phi_1 &= \square\mathcal{A}_1 + 2(\mathcal{A}_0\square\mathcal{W} - \mathcal{W}\square\mathcal{A}_0) \\ &\quad - \sum_{k=0} \frac{(-1)^k}{2 \cdot 8^k} (\square\mathcal{A}_{k+1}\square^k\bar{D}^4\bar{\mathcal{A}}_k \\ &\quad - \mathcal{A}_{k+1}\square^{k+1}\bar{D}^4\bar{\mathcal{A}}_k) = 0, \quad (35) \end{aligned}$$

and so on.

At present we do not know how to explicitly solve this set of constraints and to find a closed expression for the Lagrangian densities $\mathcal{A}_0, \bar{\mathcal{A}}_0$. We can only recover the general solution by iterations. E.g.,

$$\begin{aligned} \mathcal{A}_0 &= \mathcal{W}^2 + \mathcal{A}_0^{(4)} + \mathcal{A}_0^{(6)} + \mathcal{A}_0^{(8)} + \dots, \\ \mathcal{A}_0^{(4)} &= \frac{1}{2}\mathcal{W}^2\bar{D}^4\bar{\mathcal{W}}^2, \dots \quad (36) \end{aligned}$$

The action, up to the 10th order in $\mathcal{W}, \bar{\mathcal{W}}$, was found in [16]. It turned out to coincide, at least up to the 8th order, with the action deduced in [17] from the requirements of self-duality and invariance under nonlinear shifts of $\mathcal{W}, \bar{\mathcal{W}}$ (the c, \bar{c} transformations in our notation). This is an indication that the full $N = 4 \rightarrow N = 2$ BI action is also self-dual like its $N = 2 \rightarrow N = 1$ prototype [3].

2.4 $N = 8 \rightarrow N = 4, d = 4$ BI theory. No manifestly $N = 4$ supersymmetric off-shell actions are known for $N = 4, d = 4$ Maxwell theory, so no such actions can be defined for the BI deformations of the latter. The best what one can hope to gain is $N = 4$ BI actions with the $N = 1, N = 2$ [1] or at most $N = 3$ [5] manifest off-shell SUSYs. However, it is still possible to derive the superfield equations of motion of $N = 4$ BI theory, in a manifestly $N = 4$ supersymmetric form and with one extra nonlinearly realized $N = 4$ SUSY, within the nonlinear realizations formalism applied to the following $N = 8, D = 4$ superalgebra [15]:

$$\begin{aligned} \{Q_{\alpha}^i, \bar{Q}_{\dot{\alpha}j}\} &= \{S_{\alpha}^i, \bar{S}_{\dot{\alpha}j}\} = 2\delta_j^i P_{\alpha\dot{\alpha}}, \\ \{Q_{\alpha}^i, S_{\beta}^j\} &= \varepsilon_{\alpha\beta} Z^{ij} \quad (37) \end{aligned}$$

$$\bar{Z}_{ij} = (Z^{ij})^* = \frac{1}{2}\varepsilon_{ijkl}Z^{kl}. \quad (38)$$

The basic Goldstone superfield supporting partial breakdown of this SUSY down to $N = 4, d = 4$ SUSY $\propto (Q, P)$ is an $N = 4$ superfield W_{ik} , $\bar{W}^{ij} = \frac{1}{2}\epsilon^{ijkl}W_{kl}$, associated with the generator Z^{ik} . One also introduces spinor Goldstone superfields $\psi_i^\alpha, \bar{\psi}^{\dot{\alpha}i}$ associated with S_α^i and $\bar{S}_{\dot{\alpha}j}$. The covariant superfield equations of the $N = 8 \rightarrow N = 4$ super BI theory read

$$(a) \quad \psi_{\alpha i} + \frac{2i}{3}\mathcal{D}_\alpha^j W_{ij} = 0,$$

$$(b) \quad \mathcal{D}_\alpha^k W_{ij} - \frac{1}{3}[\delta_i^k \mathcal{D}_\alpha^m W_{mj} - (i \leftrightarrow j)] = 0 \quad (39)$$

(plus their c.c.). Here $\mathcal{D}_\alpha^j, \bar{\mathcal{D}}_{\dot{\alpha}}^{\dot{j}}$ are the appropriate covariantization of the flat $N = 4$ spinor derivatives. They nonlinearly depend on $\psi_i^\alpha, \bar{\psi}_i^{\dot{\alpha}}$ which can be covariantly expressed through derivatives of W_{kl} by eq. (39a). Eq. (39b) is a covariantization of the standard superfield constraints of the on-shell $N = 4$ Maxwell theory [18], and it contains the dynamical equations for the component fields. One of them is a disguised form of the BI equations. The full set of equations is a manifestly worldvolume supersymmetric form of the equations of the gauge-fixed D3-superbrane in $D = 10$, with 6 physical bosonic fields of $N = 4$ Goldstone-Maxwell multiplet being transverse brane coordinates.

As was already mentioned, one cannot hope to construct an off-shell $N = 4$ supersymmetric superfield action for this super BI system, since no such action exists even for the ordinary $N = 4$ gauge theory. However, for the first non-trivial term in the BI action, the quartic term $\sim F^{\alpha\beta}F_{\alpha\beta}\bar{F}^{\dot{\alpha}\dot{\beta}}\bar{F}_{\dot{\alpha}\dot{\beta}}$, $N = 4$ completions with $N = 1$ and $N = 2$ off-shell supersymmetries are known. These were given, respectively, in terms of $N = 1$ superfields [1] and $N = 2$ projective superfields [19]. Here we present this completion in terms of off-shell $N = 2$ harmonic superfields [20].

In the harmonic superspace (HSS) description [21,22], $N = 4$ vector multiplet is represented by the standard gauge $N = 2$ superfield strength $\mathcal{W}(z), \bar{\mathcal{W}}(z)$ and the analytic hypermultiplet superfield $q^{+a}(\zeta, u)$. Here $\{\zeta, u\} \equiv \{x^{\alpha\dot{\beta}}, \theta^{+\alpha}, \bar{\theta}^{+\dot{\alpha}}, u^{\pm i}\}$ are co-ordinates of an analytic subspace of $N = 2$ harmonic superspace

$\{z, u^{\pm i}\}$ and $u^{\pm i}, u^{+i}u_i^- = 1$, are harmonic variables parametrizing some internal 2-sphere S^2 . The indices a and i are doublet indices of two mutually commuting $SU(2)$ groups, $a = 1, 2; i = 1, 2$. Further details can be found in [21,22]. The free $N = 4$ Maxwell theory action is given by

$$S_{free}^{N=4} = \frac{1}{8} \int d\zeta_L \mathcal{W}^2 + \text{c.c.} - \frac{1}{2} \int d\zeta^{(-4)} q^{+a} D^{++} q_a^+, \quad (40)$$

where D^{++} is the analyticity-preserving harmonic derivative and $d\zeta^{(-4)}$ is the measure of integration over the analytic superspace. Besides being manifestly $N = 2$ supersymmetric, the action (40) is invariant under one more $N = 2$ SUSY which forms $N = 4, d = 4$ SUSY together with the manifest $N = 2$ one. For our purposes it suffices to know only the on-shell form of transformations of this hidden SUSY:

$$\delta\mathcal{W} = \frac{1}{2}\bar{\epsilon}^{\dot{\alpha}a} \bar{D}_{\dot{\alpha}}^- q_a^+, \quad \delta\bar{\mathcal{W}} = \frac{1}{2}\epsilon^{\alpha a} D_{\alpha}^- q_a^+, \\ \delta q_a^\pm = \frac{1}{4}(\epsilon_a^\beta D_\beta^\pm \mathcal{W} + \bar{\epsilon}_a^{\dot{\alpha}} \bar{D}_{\dot{\alpha}}^\pm \bar{\mathcal{W}}), \quad (41)$$

where $\epsilon^{\alpha a}, \bar{\epsilon}^{\dot{\alpha}a}$ are the Grassmann transformation parameters, $D_\alpha^\pm, \bar{D}_{\dot{\alpha}}^\pm$ are harmonic projections of the flat $N = 2$ spinor derivatives and $q^{-a} \equiv D^{--} q^{+a}$, D^{--} being the second harmonic derivative. The quartic superfield term which is invariant under the transformations (41) and yields the correct quartic term $\sim F^2 \bar{F}^2$ in the component BI action (with the correct relative coefficient w.r.t. the free Maxwell action) is uniquely restored, up to terms vanishing on the free mass shell,

$$S_4^{N=4} = \frac{1}{32} \int dz du \{ \mathcal{W}^2 \bar{\mathcal{W}}^2 - 4(q^+ \cdot q^-) \times [\mathcal{W} \bar{\mathcal{W}} - \frac{1}{3}(q^+ \cdot q^-)] \}, \quad (42) \\ (q^+ \cdot q^-) \equiv q^{+a} q_a^-.$$

Here du is the measure of integration over harmonics ($\int du \cdot 1 = 1$). The problem of $N = 4$ completing of higher-order terms of the $N = 2$ BI action (even of its simplest version [14]) is technically very complicated because the form of the

hidden on-shell $N = 4$ transformations (41) is modified from order to order in superfields.

It is remarkable that in $N = 3$ HSS [23] one can construct an off-shell $N = 3$ superextension of the *full* BI action [5].

3. N=3 BORN-INFELD THEORY

3.1 Introduction. Since the $N > 2$ super BI actions are extensions of the corresponding super Maxwell actions, the necessary condition of the existence of some off-shell super BI action is the existence of such an action for the relevant free super Maxwell theory. The maximally supersymmetric off-shell formulation of $N = 4$ gauge theory is that with manifest $N = 3$ supersymmetry. It was given in [23] in the framework of $N = 3$ harmonic superspace (HSS).

In [5], starting from this formulation, we have constructed an $N = 3$ superextension of the full BI action. As distinct from the previously known $N = 1$ and $N = 2$ super BI actions, the construction of the $N = 3$ BI action is by no means a straightforward order-by-order supersymmetrization of the bosonic BI action. The main novel feature stems from the crucial property that the Grassmann-analytic gauge potentials of $N = 3$ gauge theory in $N = 3$ HSS [23] contain, besides the physical fields including the standard gauge potential A_m , also an infinite number of the auxiliary fields. Among them there is an independent bispinor field $H_{\alpha\beta} = H_{\beta\alpha}$. The correct bilinear Maxwell term in the component action arises only after elimination of this field by its algebraic equation of motion. The $N = 3$ gauge superfield strength contains the combination $V_{\alpha\beta} = \frac{1}{4}[H_{\alpha\beta} + F_{\alpha\beta}(A)]$ of the auxiliary field and the gauge field strength.

The auxiliary component $V_{\alpha\beta}$ can be interpreted as a Legendre-type transform variable for the gauge field strength $F_{\alpha\beta}(A)$. It turns out that this specific Legendre transform of the standard bosonic BI action is determined by a real function E of the single variable $a = V^2 \bar{V}^2$ where $V^2 = V^{\alpha\beta} V_{\alpha\beta}$. The problem of $N = 3$ supersymmetrization of the BI action is then reduced to the construction of self-interaction superfield terms of the order $4k$ in the auxiliary field $V_{\alpha\beta}$. All these

terms can be constructed as the appropriate powers of the off-shell $N = 3$ superfield strengths and their spinor derivatives in the framework of an analytic subspace of $N = 3$ HSS. A generic function $E(a)$ exhausts the complete set of the $SO(2)$ self-dual nonlinear extensions of the Maxwell action, the BI one being a special representative of them. All such actions can be $N = 3$ supersymmetrized off shell.

3.2 Elements of $N = 3$ harmonic formalism. The fundamental objects of the abelian $N = 3$ gauge theory are three harmonic gauge potentials living as unconstrained superfields on the $(4+6|8)$ -dimensional analytic subspace $H(4+6|8) = \{\zeta, u\}$ of $N = 3$ HSS

$$\begin{aligned} V_2^1(\zeta, u), \quad V_3^1(\zeta, u), \quad V_3^2, \\ V_2^1 = -(\widetilde{V_3^2}), \quad V_3^1 = (\widetilde{V_3^1}). \end{aligned} \quad (43)$$

The definition of the generalized conjugation \sim preserving $N = 3$ Grassmann harmonic analyticity and the precise content of the analytic coordinate set $\{\zeta, u\}$ can be found in [23,5]. The potentials undergo abelian gauge transformations with a real analytic parameter $\lambda(\zeta, u)$:

$$\delta V_2^1 = iD_2^1 \lambda, \quad \delta V_3^1 = iD_3^1 \lambda, \quad \delta V_3^2 = iD_3^2 \lambda. \quad (44)$$

The potential V_3^1 can be consistently expressed in terms of the two remaining ones by imposing the conventional constraint

$$\hat{V}_3^1 \equiv D_2^1 V_3^2 - D_3^2 V_2^1. \quad (45)$$

The free $N = 3$ gauge theory action has the following form:

$$\begin{aligned} S_2(V_2^1, V_3^2) = & -\frac{1}{4f^2} \int d\zeta_{(11)}^{(33)} du [V_3^2 D_3^1 V_2^1 \\ & + \frac{1}{2}(D_2^1 V_3^2 - D_3^2 V_2^1)^2], \end{aligned} \quad (46)$$

where the analytic superspace integration measure $d\zeta_{(11)}^{(33)} du = d^4 x_A d^8 \theta_A^{(33)} du$ is defined in [23,5] and we have introduced the coupling constant f of dimension -2 , so that $[V_2^1] = -2$ and the gauge field strength is dimensionless. Besides an infinite number of gauge components accounted for by the gauge freedom (44), the gauge potentials possess an infinite number of the auxiliary field components. The latter disappear only

on the mass shell defined by the free equations of motion following from (46):

$$\begin{aligned} F_{23}^{11} &= D_3^1 V_2^1 - D_2^1 \hat{V}_3^1 = 0, \\ F_{33}^{12} &= D_3^2 \hat{V}_3^1 - D_3^1 V_3^2 = 0. \end{aligned} \quad (47)$$

For our further purposes it will be important to know the full structure of the bosonic $SU(3)$ singlet sector in the component expansion of the off-shell analytic potentials V_2^1 and V_3^2 in the WZ gauge. A simple analysis yields

$$\begin{aligned} v_2^1 &= \theta_2^\alpha \bar{\theta}^{1\dot{\beta}} A_{\alpha\dot{\beta}} + i(\theta_2)^2 \bar{\theta}^{1(\dot{\alpha}} \bar{\theta}^{2\dot{\beta})} \bar{H}_{\dot{\alpha}\dot{\beta}} \\ &+ i(\theta_2)^2 (\bar{\theta}^1 \bar{\theta}^2) C, \end{aligned} \quad (48)$$

$$\begin{aligned} v_3^2 &= \theta_3^\alpha \bar{\theta}^{2\dot{\beta}} A_{\alpha\dot{\beta}} - i\theta_2^{(\alpha} \theta_3^{\beta)} (\bar{\theta}^2)^2 H_{\alpha\beta} \\ &- i(\theta_2 \theta_3) (\bar{\theta}^2)^2 C, \end{aligned} \quad (49)$$

where $H_{\alpha\beta} = H_{\beta\alpha}$, $\bar{C} = C$, and the spinor representation for the gauge field strength was used

$$\begin{aligned} F_{\alpha\beta}(A) &\equiv \partial_{(\alpha}^{\dot{\beta}} A_{\beta)\dot{\beta}}, \quad \bar{F}_{\dot{\alpha}\dot{\beta}}(A) \equiv \partial_{(\dot{\alpha}}^{\beta} A_{\beta)\dot{\beta}}, \\ \partial_{\alpha}^{\dot{\beta}} \bar{F}_{\dot{\beta}\dot{\alpha}} - \partial_{\dot{\alpha}}^{\beta} F_{\beta\alpha} &= 0. \end{aligned} \quad (50)$$

We observe that the auxiliary dimensionless symmetric tensor and scalar fields $H_{\alpha\beta}$ and C are present in the off-shell $SU(3)$ singlet sector in parallel with the gauge potential $A_{\alpha\dot{\beta}}$ and its covariant field strength. The fields $H_{\alpha\beta}, \bar{H}_{\dot{\alpha}\dot{\beta}}$ play a crucial role in constructing $N = 3$ supersymmetric BI action.

The gauge fields part of the off-shell super $N = 3$ Maxwell component Lagrangian corresponding to (46) is

$$\begin{aligned} L_2(F, H, C) &= \frac{1}{16f^2} [H^2 + \bar{H}^2 \\ &- 6(\bar{H}\bar{F} + HF) + F^2 + \bar{F}^2 + 8C^2]. \end{aligned} \quad (51)$$

Eliminating the auxiliary fields $H_{\alpha\beta}, \bar{H}_{\dot{\alpha}\dot{\beta}}, C$ by their algebraic equations of motion

$$H_{\alpha\beta} = 3F_{\alpha\beta}, \quad \bar{H}_{\dot{\alpha}\dot{\beta}} = 3\bar{F}_{\dot{\alpha}\dot{\beta}}, \quad C = 0, \quad (52)$$

we arrive at the standard Maxwell action

$$L_2(F) = -\frac{1}{2f^2}(F^2 + \bar{F}^2) = -\frac{1}{4f^2}\mathcal{F}^{mn}\mathcal{F}_{mn}, \quad (53)$$

where $\mathcal{F}_{mn} = \partial_m A_n - \partial_n A_m$.

The basic building-blocks of the $N = 3$ BI action are the analytic superfield strengths. Like in the $N = 2$ gauge theory in $N = 2$ HSS [24], one firstly defines the non-analytic abelian connections V_1^2, V_2^3 via the harmonic zero-curvature equations

$$D_2^1 V_1^2 - D_1^2 V_2^1 = 0, \quad D_3^2 V_2^3 - D_2^3 V_3^2 = 0, \quad (54)$$

where $V_2^3 = -\widetilde{V}_1^2$, $\delta V_2^3 = iD_2^3 \lambda$, $\delta V_1^2 = iD_1^2 \lambda$ and the explicit form of the harmonic derivatives is given in [5]. Then the mutually conjugated Grassmann-analytic off-shell superfield strengths of the $N = 3$ Maxwell theory are constructed as follows [25]:

$$W_{23} = \frac{1}{4}(\bar{D}_3)^2 V_2^3, \quad \bar{W}^{12} = -\frac{1}{4}(D^1)^2 V_1^2. \quad (55)$$

These off-shell superfield strengths satisfy the following Grassmann analyticity conditions:

$$\begin{aligned} \bar{D}_{2\dot{\alpha}} W_{23} &= \bar{D}_{3\dot{\alpha}} W_{23} = D_{\alpha}^1 W_{23} = 0, \\ D_{\alpha}^1 \bar{W}^{12} &= D_{\alpha}^2 \bar{W}^{12} = \bar{D}_{3\dot{\alpha}} \bar{W}^{12} = 0 \end{aligned} \quad (56)$$

and harmonic differential conditions

$$D_3^2 W_{23} = 0, \quad D_2^1 \bar{W}^{12} = 0. \quad (57)$$

We shall need the full off-shell $SU(3)$ singlet component structure of W_{23}, \bar{W}^{12} . The explicit expressions for relevant parts of the latter read

$$w_{23} = i\theta_2^\alpha \theta_3^\beta V_{\alpha\beta} - (\theta_2)^2 \theta_3^\alpha \bar{\theta}^{2\dot{\beta}} \partial_{\dot{\beta}}^\beta V_{\alpha\beta}, \quad (58)$$

$$\bar{w}^{12} = i\bar{\theta}^{1\dot{\alpha}} \bar{\theta}^{2\dot{\beta}} \bar{V}_{\dot{\alpha}\dot{\beta}} + (\bar{\theta}^2)^2 \theta_2^\alpha \bar{\theta}^{1\dot{\beta}} \partial_{\dot{\beta}}^{\dot{\alpha}} V_{\alpha\dot{\beta}}, \quad (59)$$

where

$$V_{\alpha\beta} = \frac{1}{4}(H_{\alpha\beta} + F_{\alpha\beta}), \quad \bar{V}_{\dot{\alpha}\dot{\beta}} = \overline{(V_{\alpha\beta})}.$$

One can directly check that w_{23}, \bar{w}^{12} on their own obey the off-shell conditions (56) and (57).

The free Maxwell Lagrangian (51) (with $C = 0$), being rewritten through the newly introduced auxiliary fields $V_{\alpha\beta}, \bar{V}_{\dot{\alpha}\dot{\beta}}$, reads

$$\begin{aligned} L_2(F, H, 0) &\equiv B_2(F, V) = \frac{1}{f^2} [V^2 + \bar{V}^2 \\ &- 2(VF + \bar{V}\bar{F}) + \frac{1}{2}(F^2 + \bar{F}^2)]. \end{aligned} \quad (60)$$

The algebraic equations of motion for $V_{\alpha\beta}, \bar{V}_{\dot{\alpha}\dot{\beta}}$ giving rise to the standard Lagrangian (53) are simply

$$V_{\alpha\beta} = F_{\alpha\beta} , \quad \bar{V}_{\dot{\alpha}\dot{\beta}} = \bar{F}_{\dot{\alpha}\dot{\beta}} . \quad (61)$$

From the above discussion one infers two important properties of the off-shell description of $N = 3$ gauge theory in $N = 3$ HSS having no direct analogs in the $N = 1$ and $N = 2$ cases. First, the free Maxwell component Lagrangian appears in the unusual forms (51) or (60), while its standard form is recovered only after eliminating the auxiliary fields $V_{\alpha\beta}, \bar{V}_{\dot{\alpha}\dot{\beta}}$ by their linear algebraic equations of motion (61). Secondly, the off-shell superfield strengths contain just these tensor auxiliary fields, but not the ordinary gauge field strengths $F_{\alpha\beta}, \bar{F}_{\dot{\alpha}\dot{\beta}}$.

These surprising features suggest a non-standard approach to constructing nonlinear and non-polynomial superextensions of the off-shell $N = 3$ Maxwell theory. One should modify (51) by proper terms which are nonlinear (and/or non-polynomial) in the auxiliary fields $V_{\alpha\beta}, \bar{V}_{\dot{\alpha}\dot{\beta}}$, such that nonlinearities in $F_{\alpha\beta}, \bar{F}_{\dot{\alpha}\dot{\beta}}$ inherent in the BI action are regained as the result of eliminating these auxiliary fields by their *nonlinear* equations of motion. Then one can hope to $N = 3$ supersymmetrize the terms nonlinear in $V_{\alpha\beta}, \bar{V}_{\dot{\alpha}\dot{\beta}}$ with the help of the above superfield strengths W_{23}, \bar{W}^{12} which contain just these auxiliary fields.

3.3 $N = 3$ BI action. Without entering into details, the modification of the free Maxwell Lagrangian (60), such that it becomes the correct bosonic BI Lagrangian,

$$\begin{aligned} L_{BI}(F, \bar{F}) &= \frac{1}{f^2} \left[1 - \sqrt{-\det(\eta_{mn} + \mathcal{F}_{mn})} \right] \\ &\equiv \frac{1}{f^2} [1 - Q(\varphi, \bar{\varphi})] , \quad Q(\varphi, \bar{\varphi}) = \sqrt{1 + X} , \quad (62) \\ \varphi &= F^2 , \quad \bar{\varphi} = \bar{F}^2 , \\ X(\varphi, \bar{\varphi}) &\equiv (\varphi + \bar{\varphi}) + (1/4)(\varphi - \bar{\varphi})^2 , \quad (63) \end{aligned}$$

after elimination of the auxiliary fields $V_{\alpha\beta}, \bar{V}_{\dot{\alpha}\dot{\beta}}$ by their algebraic equations of motion, is as follows [5]

$$B(F, V) = B_2(F, V) + \frac{1}{f^2} E(V^2, \bar{V}^2)$$

$$\begin{aligned} &= \frac{1}{f^2} [\nu + \bar{\nu} - 2(VF + \bar{V}\bar{F}) \\ &\quad + \frac{1}{2}(\varphi + \bar{\varphi}) + E(\nu\bar{\nu})] . \quad (64) \end{aligned}$$

Here $\nu \equiv V^2, \bar{\nu} \equiv \bar{V}^2$ and $E(\nu\bar{\nu})$ is a real function of the single argument $\nu\bar{\nu} \equiv a$ defined by the following equations

$$\begin{aligned} E(a) &= 2[2t^2(a) + 3t(a) + 1] , \quad E(0) = 0 , \quad (65) \\ t^4(a) + t^3(a) - \frac{1}{4}a &= 0 , \quad t(0) = -1 . \quad (66) \end{aligned}$$

Note that the generic choice of the function $E(a)$ corresponds to a wide class of self-dual (by Legendre transformation) extensions of Maxwell action, the BI one being merely a particular case of these nonlinear actions.¹

The problem of constructing a manifestly $N = 3$ supersymmetric superfield action which would yield, in the bosonic sector, the F, V form (64) of the BI action amounts to setting up a collection of superfield monomials which extend the appropriate terms in the power expansion of the function $E(V^2\bar{V}^2)$ defined in (65), (66).

Given the function $E(V^2\bar{V}^2)$, we introduce the new function $\hat{E}(V^2\bar{V}^2)$ by

$$E(V^2\bar{V}^2) = \frac{1}{2} V^2\bar{V}^2 \hat{E}(V^2\bar{V}^2) , \quad (67)$$

with $\hat{E}(a) = 1 - a/4 + O(a^2)$. Then the whole sequence of higher order terms in the $N = 3$ generalization of the BI-action can be written as a closed expression in the analytic superspace,

$$S_E = \frac{1}{32f^2} \int dud\zeta \zeta^{(33)} (W_{23})^2 (\bar{W}^{12})^2 \hat{E}(A) . \quad (68)$$

Here A is the following real analytic superfield:

$$\begin{aligned} A &= \frac{1}{2^{11}} (D^1)^2 (\bar{D}_3)^2 [D^{2\alpha} W_{12} D_\alpha^2 W_{12} \\ &\quad \times \bar{D}_{2\dot{\alpha}} \bar{W}^{23} \bar{D}_2^{\dot{\alpha}} \bar{W}^{23}] = V^2\bar{V}^2 + \dots , \quad (69) \end{aligned}$$

$$\begin{aligned} W_{12} &= D_1^3 W_{23} = -4i \theta_1^{(\alpha} \theta_2^{\beta)} V_{\alpha\beta} + \dots , \\ \bar{W}^{23} &= -D_1^3 \bar{W}^{12} . \quad (70) \end{aligned}$$

Thus we have obtained an $N = 3$ generalization of the Born-Infeld action using the off-shell

¹See [26] for a detailed discussion of the self-duality issues in this new setting.

Grassmann-analytic potentials V_2^1 and V_3^2

$$S_{BI}^{N=3} = S_2 + S_E . \quad (71)$$

The substitution of generic function $\hat{E}(A) = 1 + O(A)$ into (68) yields $N = 3$ superextensions of a wide class of the self-dual nonlinear deformations of Maxwell theory.

Finally, we notice that the above $N = 3$ BI action is a minimal $N = 3$ extension of the bosonic BI action. It still remains to examine whether it admits a treatment as a $N = 3$ Goldstone-Maxwell superfield action describing an off-shell PBGS option $N = 6 \rightarrow N = 3, d = 4$ which could amount on shell to the option $N = 8 \rightarrow N = 4, d = 4$. By analogy to the situation with $N = 2$ BI action [14–16], one could expect that for such an interpretation to be possible the above action should be modified by some extra terms with extra x -derivatives on them.

4. CONCLUDING REMARKS

In conclusion, it is worth mentioning a few important unsolved problems in supersymmetric BI theories.

- Constructing a BI deformation of the off-shell $N = (1, 0), d = 6$ super Maxwell action. Such super BI action should amount to a static gauge of the space-filling D5-superbrane action in a flat Minkowski background.
- Adapting the nonlinear realizations formalism to the case of non-abelian BI actions. This would seemingly require introducing the notion of generalized supersymmetry algebras incorporating the non-abelian gauge group structure.
- Constructing superconformally invariant versions of $N = 2$ and $N = 3$ super BI actions reviewed in this article. Such modifications could play an important role in the context of the AdS/CFT correspondence [27]–[30], providing the PBGS form of the effective worldvolume actions of D3-superbranes on superbackgrounds with the $\text{AdS}_n \times S^m$ -type bosonic body ($\text{AdS}_5 \times S^1$

in the $N = 2$ case and $\text{AdS}_5 \times S^5$ in the $N = 3$ and $N = 4$ cases).

Note that an $N = 4$ completion of the F^4 term [31] of the $N = 2$ superconformal BI action in $N = 2$ HSS was found in a recent paper [32]. It radically differs from its non-conformal counterpart (42):

$$S_4^{conf} \sim \int dz du \left\{ \ln \frac{\mathcal{W}}{\Lambda} \ln \frac{\bar{\mathcal{W}}}{\Lambda} + (X - 1) \frac{\ln(1 - X)}{X} + [\text{Li}_2(X) - 1] \right\} . \quad (72)$$

Here

$$X = -2 \frac{q^+ \cdot q^-}{\mathcal{W}\bar{\mathcal{W}}} , \quad (73)$$

and

$$\text{Li}_2(X) = - \int_0^X \frac{\ln(1 - t)}{t} dt = \sum_{n=1}^{\infty} \frac{1}{n^2} X^n$$

is Euler dilogarithm (Λ is an arbitrary scale). The action (72) is invariant under both $N = 2$ superconformal group and $N = 4$ supersymmetry (41), hence it is invariant under the whole $N = 4$ superconformal group.

As distinct from (42), the contributions to the component F^4 term come from both the pure $\mathcal{W}, \bar{\mathcal{W}}$ and mixed $\mathcal{W}, \bar{\mathcal{W}}, q^\pm$ pieces of (72). It reads

$$\sim \frac{F^2 \bar{F}^2}{(|\varphi|^2 + f^{ia} f_{ia})^2} , \quad (74)$$

where $\varphi(x), \bar{\varphi}(x)$ ($\langle \varphi \rangle \neq 0$) and $f^{ia}(x)$ are physical bosonic fields of the vector $N = 2$ multiplet and hypermultiplet. Together they form a 6-dimensional multiplet of the R -symmetry group $SU(4)$ of $N = 4$ SUSY. The $SU(4)$ invariant square of these fields in the denominator of (74) ensures the scale and conformal invariance of the corresponding x -space action and can be identified, from the $\text{AdS}_5 \times S^5$ D3-superbrane perspective [27], with the fifth (radial) co-ordinate of AdS_5 .

It would be extremely interesting to find a superconformally invariant version (if existing) of the $N = 3$ BI action (71). Such an action is expected to be unique and to provide a manifestly

off-shell $N = 3$ supersymmetric superfield form of the abelian D3-superbrane action on $\text{AdS}_5 \times S^5$. It is very important to know this hypothetical maximally worldvolume supersymmetric form of the D3-brane action both for further clarifying the AdS/CFT correspondence and, as a closely related goal, for exploring the precise structure of the quantum low-energy effective action in $N = 4$ SYM theory.

Acknowledgements

This work was supported by INTAS grant No 00-254, RFBR grant No 99-02-18417, RFBR-CNRS grant No 01-02-22005 and a grant of Bogoliubov-Infeld Programme.

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